AP Environmental Science
Basic Math Skills and Unit Conversions

- The APES curriculum requires you to perform calculations **in every unit**.
- Sometimes the math is easy enough to solve in your head in a single step and sometimes it will require multiple steps.
- Regardless, you are required to **SHOW ALL WORK (including units)** in order to receive credit for calculation problems on labs, tests, and the AP Exam.
- I will guide you through the more complex calculations throughout the year, but this presentation shows the basic foundational math you are expected to know upon entering the course.
- Be sure you can do all the practice problems to prepare for a quiz the first week of class (no calculator).

Writing in Scientific Notation

While not required doing APES calculations in scientific notation can be much simpler and can save you a lot of precious time on the AP Exam. For that reason, I recommend it and most of the math I demonstrate in class will involve numbers written in scientific notation.

* Scientific notation is a way of writing very large and very small numbers.
* It involves writing the number as a number between 1 & 10, multiplied by 10 raised to an exponent.
* The exponent in scientific notation represents the number of times the decimal point must be moved to produce a number between 1 & 10.

Ex 1. Converting 6,700,000 to scientific notation involves moving the decimal to the left 6 spaces and would be written as 6.7×10^6

Ex 2. Converting 0.0000670 to scientific notation involves moving the decimal to the right 5 space and would be written as 6.7×10^5

Adding and Subtracting Numbers in Scientific Notation

There are some basic rules for doing mathematical operations with numbers written in scientific notation. You must memorize these, if you don't already know them.

To add or subtract numbers in scientific notation, the exponents must be the same and then you just add or subtract the coefficients. Then, replace the exponential power.

Ex 1. $4.5 \times 10^5 + 2.2 \times 10^5 = 6.7 \times 10^5$

Ex 2. $4.5 \times 10^5 - 2.2 \times 10^5 = 2.3 \times 10^5$

Multiplying and Dividing in Scientific Notation

There are some basic rules for doing mathematical operations with numbers written in scientific notation. You must memorize these, if you don't already know them.

To multiply or divide numbers in scientific notation, the exponents do NOT need to be the same. Multiply or divide the coefficients and then ADD the exponents for multiplication operations and SUBTRACT the exponents for division operations.

Ex 1. $2.0 \times 10^4 \times 3.0 \times 10^7 = (2.0 \times 3.0) = 6.0$ and $(4 + 7) = 11$

So, the answer $= 6.0 \times 10^{11}$

Ex 2. $8.0 \times 10^9 / 4.0 \times 10^3 = 2.0 \times 10^6$

DO NOW: Practice Manipulating and Calculating using Scientific Notation

1. Write the following numbers in proper scientific notation:
   a. Thirty-four thousand =
   b. 0.026 =
   c. 5250 =
2. Convert the following to regular notation:
   a. 4.25×10^{3} =
   b. 8.5×10^{-4}=
3. Use scientific notation solve the following problems:
   a. $(4.0 \times 10^{-6})\ (2.0 \times 10^{3})$ =
   b. $\dfrac{(3.0 \times 10^{6})}{(1.0 \times 10^{11})}$ =
   c. $(4.0 \times 10^{2}) + (6.0 \times 10^{3})$ =

Mathematical Operations involving Conversions and Percent

How many miles can you travel at a steady rate of 45 mph for 4 hours?

$$\frac{45\ \text{mi}}{1\ \text{hr}} \times 4\ \text{hr} = 180\ \text{mi}$$

How many minutes will it take to travel 300 miles of 50 mph?

$$\frac{300\ \text{mi}}{50\ \text{mi}} \times \frac{1\ \text{hr}}{1\ \text{hr}} \times 60\ \text{min} = 360\ \text{min}$$

The number of people in developed countries, if 20% of world pop (7.4 billion)?

$$\frac{20}{100} \times 7.4 \times 10^9\ \text{people} = 14.8 \times 10^8 = 1.48 \times 10^9\ \text{people}$$

DO NOW: Practice Conversions and Percent

4. Traveling 65 mph, how long in minutes is the 624-mile drive?

5. Determine 25% of a $42.50 bill so that you can give a generous tip.

6. Developers purchase 40% of a natural area, leaving 2200 acres untouched. How many acres are to be developed?

7. Sixteen million is what percentage of the U.S. population of 320 million?

8. Calculate the annual percent growth for a country with 8 million inhabitants, in a year in with 120,000 births and 80,000 deaths.

Calculations using the Metric System

Metric Prefixes
in intervals of 1000 (10^3)

← smaller unit prefixes				base unit	larger unit prefixes →			
pico-	nano-	micro-	milli-	*unit*	kilo-	mega-	giga-	tera-
(p) 10^{-12} 1 pico- = 10^{-12} units **or** 10^{12} pico- = 1 unit	(n) 10^{-9} 1 nano- = 10^{-9} units **or** 10^{9} nano- = 1 unit	(μ) 10^{-6} 1 micro- = 10^{-6} units **or** 10^{6} micro- = 1 unit	(m) 10^{-3} 1 milli- = 10^{-3} units **or** 10^{3} milli- = 1 unit	(--) 10^{0} EXAMPLE ↓	(k) 10^{3} 1 kilo- = 10^{3} units (thousand)	(M) 10^{6} 1 mega- = 10^{6} units (million)	(G) 10^{9} 1 giga- = 10^{9} units (billion)	(T) 10^{12} 1 tera- = 10^{12} units (trillion)
pW = 10^{-12} W	nW = 10^{-9} W	μW = 10^{-6} W	mW = 10^{-3} W	**watt** **(W)**	kW = 10^{3} W	MW = 10^{6} W	GW = 10^{9} W	MW = 10^{12} W

Base Unit Examples

Mass: grams (g)	Time: seconds (s)	Power: watt (W)	Pressure: pascal (Pa)
Length: meters (m)	Energy/Work: joule (J)	Force: newton (N)	Current: ampere (A)

Additional Metric Prefixes between 10^{-3} ← base unit → 10^{3}

← smaller unit prefixes			base unit	larger unit prefixes →		
milli-	centi-	deci-	*unit*	deca-	hecto-	kilo-
(m) 10^{-3} 1 milli- = 10^{-3} units **or** 10^{3} milli- = 1 unit	(c) 10^{-2} 1 centi- = 10^{-2} units **or** 10^{2} centi- = 1 unit	(d) 10^{-1} 1 deci- = 10^{-1} units **or** 10^{1} centi- = 1 unit	(--) 10^{0} EXAMPLE ↓	(da) 10^{1} 1 deca- = 10^{1} units (ten)	(h) 10^{2} 1 hecto- = 10^{2} units (hundred)	(k) 10^{3} 1 kilo- = 10^{3} units (thousand)
mm = 10^{-3} m	cm = 10^{-2} m	dm = 10^{-1} m	**meter (m)**	dam = 10^{1} m	hm = 10^{2} m	k = 10^{3} m

of these prefixes, we will likely only use centi-

Calculations Involving Metric Conversions

You collect 1250 g of soil for analysis. What mass is this in kg?

$$\frac{1250\ \text{g}}{} \left| \frac{1\ \text{kg}}{1000\ \text{g}} \right. = 1.25\ \text{kg of soil}$$

You prepare a salt solution of 32.7 g/L. What is the concentration is in g/mL?

$$\frac{32.7\ \text{g}}{} \left| \frac{1\ \text{L}}{1000\ \text{mL}} \right. = 0327\ \text{g/mL} = 3.27 \times 10^{2}\ \text{g/mL salt solution}$$

You purchase a tarp that is 4500 cm^2 in area. How many m^2 will this cover?

$$\frac{4500\ \text{cm}^2}{} \left| \frac{(1)^2\ \text{m}^2}{(100)^2\ \text{cm}^2} \right. = \frac{4500\ \text{m}^2}{10000} = 0.45\ \text{m}^2\ \text{coverage}$$

DO NOW: Practice Metric Conversions

9. You hammer a stake 6850 mm into the ground. What depth in cm is the stake?

10. What concentration (g/L) of homemade hummingbird nectar have you prepared if you dissolve 0.56 kg sugar in 1L of water?

11. 2 µg/kg of clay is present in your soil. How much clay is this in ng/kg?

12. Earth's freshwater volume is 3.45×10^8 km^3. How much is this in cubic meters?

Unitless Conversions

Parts and Portions

smaller portions ⟷ larger portions

Parts per trillion (ppt)	Parts per billion (ppb)	Parts per million (ppm)	Parts per thousand (‰) [aka per-mil]	Parts per hundred (%) [aka percent]
$\frac{1}{1{,}000{,}000{,}000{,}000}$	$\frac{1}{1{,}000{,}000{,}000}$	$\frac{1}{1{,}000{,}000}$	$\frac{1}{1{,}000}$	$\frac{1}{100}$
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}

Converting numbers with units to unitless

Ex. What percent of 1 foot is 3 inches?

$$\frac{3\text{ in}}{1\text{ ft}} = \frac{3\text{ in}}{12\text{ in}} = \frac{1}{4} = \frac{25}{100} = 25\%$$

Must have the same base unit in the numerator and denominator to calculate a unitless quantity, because the units must cancel. There are 12 in in a 1 ft, so 3 inches per 12 inches is 1 part per 4. Scale up the numbers to 25 parts per hundred, where “parts per hundred” is also know as “percent”.

Converting from Metric Units with to Unitless
This will require you KNOW your metric prefixes!

What is the concentration (in ppb) of copper if 150 ug of Cu dissolves in 3 mL of water?

Cu concentration in metric units

$$\frac{150 \text{ ug Cu}}{3 \text{ mL}} = 50 \text{ ug/mL Cu}$$

Need same base unit in numerator and denominator
(1 mL H_2O = 1 g mass)

$$50 \frac{\text{ug}}{\text{mL}} = 50 \frac{10^{-6} \cancel{\text{g}}}{1 \cancel{\text{g}}} = 50 \bullet 10^{-6} = \frac{50}{10^6} = 50 \text{ ppm Cu}$$

The concentration unit ug/g is the same as one part per million (ppm). Now convert to ppb.

$$\frac{50}{10^6} = \frac{x}{10^9} \qquad x = \underline{50{,}000 \text{ ppb Cu}}$$

DO NOW: Practice Unit to Unitless Conversion

$$\frac{ng}{mg} = \frac{10^{-9}\,\cancel{g}}{10^{-3}\,\cancel{g}} = \frac{1}{(10^{-3})(10^{9})} = \frac{1}{10^{6}} = ppm$$

13. $\frac{ug}{kg} =$

14. $\frac{ng}{mg} =$

15. $\frac{ng}{g} =$

16. $\frac{ng}{kg} =$

17. $\frac{mg}{g} =$

18. $\frac{mg}{L} =$

DO NOW: Practice Calculations

19. Given 40 µg/L (aqueous solution). Convert to units of ppb.

20. What is the ppm concentration of a gas stream containing 3.2% oxygen?

21. Dissolved oxygen (DO) analysis shows 15 mg O_2 dissolved in 450 mL of water. What is the DO in ppm?

22. Low sulfur fuels of less than 500 ppm S are being introduced for most diesel engine applications. What percent sulfur are these fuels?

DO NOW: Practice Calculations (cont.)

23. What concentration in ppm is an exhaust mixture containing 0.25% SO_2?

24. Express 49,720,000 in scientific notation.

25. Analysis of drinking water in Well A reveals Pb at a concentration of 0.04 mg/L. Well B water analysis shows one-fourth the contamination of Well A. The EPA standard for drinking water requires less than 15 ppb. Determine whether or not the wells are in compliance with the drinking water standard.

26. What is the ppm concentration of a gas stream containing 0.0012% O_2?

Practice Calculations (cont.)

27. What is the dissolved salt concentration in ppm for a solution of 3.25 g/L NaCl?

28. In 2020, atmospheric CO_2 was ~400 ppm.

a. What was the % CO_2 in that year?

b. In 1960, atmospheric CO_2 was 310 ppm. What is the average yearly rate of CO_2 increase in ppm from 1960 to 2020?

Practice Calculations (cont.)

29. Convert 383.4 g/kg to units of ppm, expressed in scientific notation.

30. Express 0.000286 in scientific notation.

31. A gas stream contains 8000 ppb Ar. What % Ar is this concentration?

32. Convert 9.4 μg/kg to units of ppm, expressed in scientific notation.